

Holographic, $\mathcal{N} = 1$ supersymmetric RG flows on M2 branes

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP09(2009)043

(<http://iopscience.iop.org/1126-6708/2009/09/043>)

[The Table of Contents](#) and [more related content](#) is available

Download details:

IP Address: 80.92.225.132

The article was downloaded on 01/04/2010 at 13:44

Please note that [terms and conditions apply](#).

Holographic, $\mathcal{N} = 1$ supersymmetric RG flows on M2 branes

Nikolay Bobev,^{a,b} Nick Halmagyi,^{c,d,e} Krzysztof Pilch^a and Nicholas P. Warner^a

^a*Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484, U.S.A.*

^b*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, U.S.A.*

^c*Laboratoire de Physique Théorique et Hautes Energies, CNRS UMR 7589, Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris Cedex 05, France*

^d*Laboratoire de Physique Théorique de l'École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France*

^e*Institut de Physique Théorique, CEA/Saclay, CNRS-URA 2306, Orme des Merisiers, F-91191 Gif sur Yvette, France*

E-mail: bobev@usc.edu, halmagyi@lpt.ens.fr, pilch@usc.edu, warner@usc.edu

ABSTRACT: We find a family of holographic $\mathcal{N} = 1$ supersymmetric RG flows on M2 branes. These flows are driven by two mass parameters from the maximally ($\mathcal{N} = 8$) supersymmetric theory and the infra-red theory is controlled by two fixed points, one with G_2 symmetry and the other with $SU(3) \times U(1)$ symmetry and $\mathcal{N} = 2$ supersymmetry. The generic flow, with unequal mass parameters, is $\mathcal{N} = 1$ supersymmetric but goes to the $SU(3) \times U(1)$ symmetric, $\mathcal{N} = 2$ supersymmetric fixed point, where the masses are equal. The only flow that goes to the G_2 symmetric point occurs when one of the mass parameters is set to zero. There is an $\mathcal{N} = 1$ supersymmetric flow from the G_2 symmetric point to the $SU(3) \times U(1)$ symmetric point and supergravity gives a prediction of $\pm \frac{1}{\sqrt{6}}$ for the anomalous dimensions of the operators that drive this flow. We examine these flows from the field theory perspective but find that one is limited to qualitative results since $\mathcal{N} = 1$ supersymmetry in three dimensions is insufficient to protect the form and dimensions of the operators involved in the flow.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence

ARXIV EPRINT: [0901.2736](https://arxiv.org/abs/0901.2736)

Contents

1	Introduction	1
2	An SU(3) invariant family of flows	3
2.1	The Bagger-Lambert-Gustavsson action in superspace	3
2.2	The family of RG flows	5
3	Mass perturbations in maximal supergravity	7
3.1	The scalars of gauged supergravity and their holographic duals	7
3.2	The scalar action of gauged supergravity	8
3.3	The supersymmetric flow equations	10
3.4	Flows near the critical points	12
4	Final comments	14
A	Projecting $\mathcal{N}=2$ to $\mathcal{N}=1$ superspace in three dimensions	15

1 Introduction

The field theory on M2 branes has always posed something of a problem in that in its simplest, most supersymmetric formulation, it is necessarily strongly coupled [1]. As a consequence, some of the non-trivial results about infra-red flows and fixed points in this theory were first obtained via holography. It was known from much older work on four-dimensional gauged supergravity [2] that the maximally supersymmetric ($\mathcal{N}=8$) theory must have two non-trivial supersymmetric infra-red fixed points corresponding to massive flows in the field theory. Some of these holographic flows were explicitly constructed in gauged supergravity [3–7] and directly in M-theory [8] (see also [9–11] for more recent work). More generally, there are quite a number of known supersymmetric compactifications of M-theory that correspond to M2 brane configurations and might therefore be incorporated into a web of RG flows that connect to the $\mathcal{N}=8$ theory. While there were some interesting parallels between the M2-brane theory and the compactification of $\mathcal{N}=4$ Yang-Mills theory, progress in this area was limited by the lack of a good field theory description on the M2 brane.

Recently our understanding of the underlying M2-brane field theory has vastly improved [12–18]. This theory may be understood in terms of an $\mathcal{N}=6$ Chern-Simons-matter theory in which the level, k , emerges from a \mathbb{Z}_k orbifold. That is, if one takes the compactifying manifold to be S^7/\mathbb{Z}_k , where the \mathbb{Z}_k acts on the Hopf fiber, then for $k > 2$ the supersymmetry is broken to $\mathcal{N}=6$ and the \mathcal{R} -symmetry is broken from $\text{SO}(8)$ to $\text{SO}(6)$. The coupling of this field theory may be thought of as k^{-1} , and so it is weakly coupled for

large k . For $k = 1, 2$ the full $\mathcal{N} = 8$ supersymmetry and $SO(8)$ \mathcal{R} -symmetry is preserved (but not manifestly within the ABJM formalism) but the theory is strongly coupled. This formulation has enabled one to re-examine and understand the supergravity flows from the field-theory perspective [19, 20].

In this paper we will exhibit and study a family of $\mathcal{N} = 1$ supersymmetric RG flows using the maximally supersymmetric $\mathcal{N} = 8$ theory. This family is controlled by two infrared fixed points:

- I** A fixed point with $\mathcal{N} = 1$ supersymmetry and a global G_2 symmetry. The flow corresponds to turning on a single mass parameter, with the remaining massless bosons and fermions on the M2 brane transforming in the $\mathbf{7}$ of G_2 .
- II** A fixed point with $\mathcal{N} = 2$ supersymmetry, a $U(1)$ \mathcal{R} -symmetry and a global $SU(3)$ symmetry. The flow corresponds to turning on two equal mass parameters, with the six remaining massless bosons and fermions on the M2 brane transforming in the $\mathbf{3} + \bar{\mathbf{3}}$ of $SU(3)$.

This family of flows is driven by two *independent* mass parameters arranged so that when one of them vanishes the theory flows in a G_2 invariant manner to fixed point **I** and when they are equal the theory flows in a $SU(3) \times U(1)$ invariant manner to fixed point **II**. We will show that the dominant fixed point is the “lower” fixed point, **II**. That is, the generic flow with two unequal masses flows to fixed point **II** where the two masses become equal and one only reaches fixed point **I** if one of the mass parameters is exactly zero. If one of the mass parameters is tiny compared to the other then the flow can approach fixed point **I** arbitrarily closely before diverting to fixed point **II**. There is also a flow directly from fixed point **I** to fixed point **II**, and the supergravity gives a prediction of $\pm \frac{1}{\sqrt{6}}$ for the anomalous dimensions of the relevant operators that drive this flow.

We also examine these flows using the field theory. Indeed, the $\mathcal{N} = 2$ supersymmetric flow and fixed point has been extensively studied in [19, 20] and the results have been matched directly with supergravity [21]. The field theory description of the family of $\mathcal{N} = 1$ flows is rather more qualitative. This is because the $\mathcal{N} = 2$ flows are driven by F-terms and are thus based upon operators whose dimensions and interactions are protected by the $U(1)$ \mathcal{R} -symmetry. In the $\mathcal{N} = 1$ flows, the action consists only of D-terms and these generally undergo non-trivial renormalizations. We discuss the usual procedure of integrating out the massive fields and find that it gives some reasonable qualitative results that match the supergravity, but the $\mathcal{N} = 1$ supersymmetry limits the analysis significantly and does not show that generic mass perturbations flow to fixed point **II**. It also remains unclear how one might compute the anomalous dimensions of $\pm \frac{1}{\sqrt{6}}$ in the field theory. We thus have supergravity predictions that pose an interesting challenge for field theory.

In section 2 we discuss the field theory underlying the family of $\mathcal{N} = 1$ flows from the $\mathcal{N} = 8$ M2-brane theory and we present the dual supergravity analysis in section 3. We summarize our results and conclude in section 4. Some technical details of superfield expansions have been put in the appendix.

2 An SU(3) invariant family of flows

We discuss a family of $\mathcal{N} = 1$ supersymmetric flows away from the Bagger-Lambert-Gustavsson (BLG) theory, triggered by masses for two real $\mathcal{N} = 1$ superfields. It is important to state the caveat that by using the BLG theory, we are really studying two M2 branes: The gauge group of the BLG theory is $SU(2) \times SU(2)$ and so one is not exactly in the large- N limit. Nonetheless, the intuition gathered from studying deformations of the BLG action will prove to be confirmed in the gravity dual. It is also possible that some version of these dual supergravity geometries will prove useful in more general ABJM theories.

It is also important to note that, for the class of supersymmetric flows we study here, the supersymmetry will actually be completely broken in the general ABJM model. This is because the generic ABJM theory has $\mathcal{N} = 6$ supersymmetry and an $SO(6)$ \mathcal{R} -symmetry and it is only for an $SU(2) \times SU(2)$ gauge group with $k = 1, 2$ that this symmetry is enhanced to $\mathcal{N} = 8$. (The two extra supersymmetries transform as $SO(6)$ singlets.) The family of flows we consider here break the $\mathcal{N} = 8$ supersymmetry to $\mathcal{N} = 1$ while preserving the $SU(3)$ subgroup of $SO(6)$. This means that our flows do not preserve any of the $\mathcal{N} = 6$ supersymmetries of the ABJM theories and preserve one (or both) of the two $SO(6)$ singlet supersymmetries. Thus we focus on the BLG theory for which our supersymmetries are unbroken within the field theory.

2.1 The Bagger-Lambert-Gustavsson action in superspace

The BLG theory can be written in $\mathcal{N} = 2$ superspace with manifest $SU(4) \times U(1)$ symmetry [19] and in $\mathcal{N} = 1$ superspace with $SO(7)$ symmetry [22]. One quirk of the BLG formulation is that the gauge superfield \mathcal{V}_{ab} (which has a component $A_{\mu ab}$) has to be contracted in two different ways, and for this we define $\mathcal{V}^a{}_b$ and $\hat{\mathcal{V}}^a{}_b$:

$$\mathcal{V}^a{}_b = h^{ac} \mathcal{V}_{cb}, \quad \hat{\mathcal{V}}^a{}_b = f^{cda}{}_b \mathcal{V}_{cd}. \tag{2.1}$$

The tensor h^{ac} is the non-degenerate bilinear form associated with the three-algebra and the components of the tensor $f^{abc}{}_d$ are the structure constants of the three-algebra [15]. The apparently unusual fact that we need both of these contractions of the gauge field can be understood by converting to an $SU(2) \times SU(2)$ gauge theory and re-writing the theory in terms of bi-fundamental matter [17]. This description also requires the use of complex combinations of the scalar superfields. The advantage of formulating the BLG theory as a bi-fundamental gauge theory is that it leads to the ABJM generalization, which has gauge group $U(N) \times U(N)$. However the gauge index structure of these models, for $N > 2$, does not allow for local holomorphic mass terms whereas the original BLG theory does allow for such mass terms.

We therefore work with the BLG action:¹

$$S_{\text{BLG}} = k(S_{\text{CS}} + S_{\text{kin}} + W), \tag{2.2}$$

¹We will always scale fields such that the level appears as a factor multiplying the whole action.

where

$$\begin{aligned}
 S_{\text{CS}} &= \int d^3x d^4\theta \int_0^1 dt \text{tr} \left(\mathcal{V} \overline{D}^\alpha e^{it\hat{\mathcal{V}}} D_\alpha e^{-it\hat{\mathcal{V}}} \right), \\
 S_{\text{kin}} &= \int d^3x d^4\theta \overline{\mathcal{Z}}^a_A e^{-2\hat{\mathcal{V}}} \mathcal{Z}^{A,a}, \\
 W &= -\frac{1}{24} \epsilon_{ABCD} \epsilon^{abcd} \int d^3x d^2\theta \mathcal{Z}_a^A \mathcal{Z}_b^B \mathcal{Z}_c^C \mathcal{Z}_d^D + \text{c.c.}
 \end{aligned} \tag{2.3}$$

We can now decompose the $\mathcal{N}=2$ superspace expression above to $\mathcal{N}=1$ superspace. As described in the appendix, the $\tilde{\mathcal{Z}}^A$ fields are the $\mathcal{N}=1$ projections of the $\mathcal{N}=2$ superfields \mathcal{Z}^A . Using the complex structure on the $\mathcal{N}=1$ superfields

$$\begin{aligned}
 \tilde{\mathcal{Z}}^1 &= \Phi^1 + i\Phi^2, & \tilde{\mathcal{Z}}^2 &= \Phi^3 + i\Phi^4, \\
 \tilde{\mathcal{Z}}^3 &= \Phi^5 + i\Phi^6, & \tilde{\mathcal{Z}}^4 &= \Phi^7 + i\Phi^8
 \end{aligned} \tag{2.4}$$

we recover the SO(7) invariant description of BLG of Mauri-Petkou [22],

$$\begin{aligned}
 S_{\text{BLG}} &= \int d^3x d^2\theta \left(-\frac{1}{2} (D^\alpha \Phi_b^I - \epsilon^{abc} \Gamma_{ab}^\alpha \Phi_c^I)^2 - \frac{1}{8} \epsilon^{abcd} (D^\alpha \Gamma_{ab}^\beta) (D_\beta \Gamma_{\alpha cd}) \right. \\
 &\quad \left. - \frac{1}{6} \epsilon^{cda} \epsilon^{efgh} (D^\alpha \Gamma_{ab}^\beta) \Gamma_{\alpha cd} \Gamma_{\beta ef} - \frac{1}{24} \epsilon^{abcd} C_{IJKL} \Phi_a^I \Phi_b^J \Phi_c^K \Phi_d^L \right).
 \end{aligned} \tag{2.5}$$

The tensor C_{IJKL} is the self-dual SO(7) invariant tensor

$$\begin{aligned}
 C_{IJKL} &= \left(\delta_{IJKL}^{1234} + \delta_{IJKL}^{5678} + \delta_{IJKL}^{1256} + \delta_{IJKL}^{3478} + \delta_{IJKL}^{3456} + \delta_{IJKL}^{1278} \right. \\
 &\quad \left. - (\delta_{IJKL}^{1357} + \delta_{IJKL}^{2468}) + (\delta_{IJKL}^{2457} + \delta_{IJKL}^{1368}) + (\delta_{IJKL}^{1458} + \delta_{IJKL}^{2367}) + (\delta_{IJKL}^{1467} + \delta_{IJKL}^{2358}) \right).
 \end{aligned} \tag{2.6}$$

Written this way we can identify the first line in (2.7) as coming from the $\mathcal{N}=2$ D -terms in (2.2) whereas the second line in (2.7) comes from the F -terms in (2.2).

Recall from the appendix that the $\mathcal{N}=2$ vector multiplet \mathcal{V} contains an $\mathcal{N}=1$ vector multiplet Γ^α and an auxiliary real $\mathcal{N}=1$ scalar multiplet R . The D -term contributions to the superpotential are obtained by integrating out R .

More precisely, we can write the $\mathcal{N}=1$ superpotential as:

$$\begin{aligned}
 \widetilde{W}(\tilde{\mathcal{Z}}, \overline{\tilde{\mathcal{Z}}}) &= -\frac{1}{24} \epsilon^{abcd} C_{IJKL} \Phi_a^I \Phi_b^J \Phi_c^K \Phi_d^L \\
 &= \frac{1}{8} \epsilon^{abcd} \overline{\tilde{\mathcal{Z}}}_a^A \tilde{\mathcal{Z}}_b^A \overline{\tilde{\mathcal{Z}}}_c^B \tilde{\mathcal{Z}}_d^B + \frac{1}{48} \epsilon^{abcd} \epsilon_{ABCD} (\tilde{\mathcal{Z}}_a^A \tilde{\mathcal{Z}}_b^B \tilde{\mathcal{Z}}_c^C \tilde{\mathcal{Z}}_d^D + \overline{\tilde{\mathcal{Z}}}_a^A \overline{\tilde{\mathcal{Z}}}_b^B \overline{\tilde{\mathcal{Z}}}_c^C \overline{\tilde{\mathcal{Z}}}_d^D),
 \end{aligned} \tag{2.7}$$

where, as described in the appendix, the $\tilde{\mathcal{Z}}^A$ fields are the $\mathcal{N}=1$ projections of the $\mathcal{N}=2$ superfields \mathcal{Z}^A and the Φ^I are the real components of the complex fields $\tilde{\mathcal{Z}}^A$. The first (second) term in (2.7) comes from the first (second) line in (2.7) and thus come from D(F)-terms of the $\mathcal{N}=2$ action.

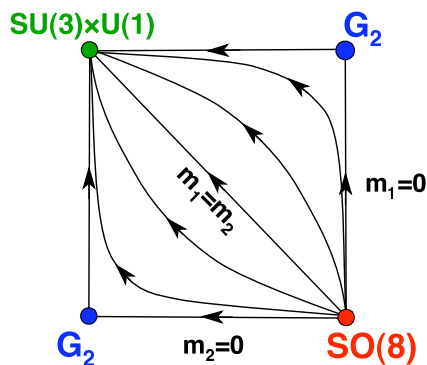


Figure 1. The pattern of RG flows to the infra-red. Starting from the $SO(8)$ -invariant fixed point the theory flows to a G_2 -invariant fixed point only if one of the masses vanishes. If both masses are non-zero, but not necessarily equal, then the theory flows to the $SU(3) \times U(1)$ -invariant fixed point where the masses become equal. The two G_2 -invariant points are equivalent and there is a flow directly from this fixed point to the $SU(3) \times U(1)$ -invariant fixed point.

2.2 The family of RG flows

We now consider the deformation of the BLG action by adding mass terms of the form:

$$\Delta W_{\text{BLG}} = \frac{1}{2}m_7 \Phi_7^2 + \frac{1}{2}m_8 \Phi_8^2. \tag{2.8}$$

If we just give a mass to one of the fields ($m_7 = 0$ or $m_8 = 0$), then we preserve G_2 symmetry. This is easily seen since the bosons are in $\mathbf{8}_v$ of $SO(8)$ and the spinors are in $\mathbf{8}_s$, so giving a supersymmetric mass to one boson and one fermion will preserve the subgroup G_2 . If we give an unequal mass to both fields, then the remaining symmetry is $SU(3)$ and if we give an equal mass to both fields the symmetry is $SU(3) \times U(1)$. With the extra $U(1)$ symmetry we preserve $\mathcal{N}=2$ supersymmetry.

We find that the IR behavior of this family of flows is best studied through the gravity dual and this is done in the next section of this paper. The picture that emerges is that if one of the masses vanishes then the theory flows to a G_2 invariant SCFT while for all other values of (m_7, m_8) the theory flows to the unique $SU(3) \times U(1)$ invariant point. In this sense the $SU(3) \times U(1)$ point is a basin of attraction for these flows.

Further, the gravity dual shows that there is a distinct RG flow from the G_2 symmetric SCFT to the $SU(3) \times U(1)$ point which preserves just $SU(3)$ along the flow. Given that such a flow lies at the boundary of the family studied here, one is led to conclude that this flow is triggered by the second mass term however it is currently difficult to perform a mapping of the operator spectrum in the field theory to the spectrum of supergravity modes. Indeed, the anomalous dimensions predicted by the supergravity suggest that this operator mapping will be rather non-trivial.

Whilst this detailed picture of the family of RG flows emerges from the gravity dual, one can also get some intuition about the family of flows by studying the field theory. From the explicit form of the $\mathcal{N}=1$ superpotential (2.7) one finds that, when $m_7 \neq m_8$, the usual techniques of integrating out these masses is problematic. For example, if we set $m_7 = 0$,

then we can seemingly integrate out Φ_8 to obtain:

$$W_{G_2} \sim (\epsilon^{abcd} C_{IJK8} \Phi_a^I \Phi_b^J \Phi_c^K)^2 + \epsilon^{abcd} \sum_{IJKL \neq 8} C_{IJKL} \Phi_a^I \Phi_b^J \Phi_c^K \Phi_d^L, \quad (2.9)$$

which has terms quartic and sextic in the remaining seven fields. Since these fields transform in the **7** of the unbroken G_2 , they must individually have equal dimension, and so one might reasonably expect that the quartic and sextic terms to have different dimensions and thus conclude that the resulting theory cannot be conformal. This is not quite accurate since $\mathcal{N} = 1$ supersymmetry in three dimensions has no \mathcal{R} -symmetry. As a result one cannot conclude that the dimension of monomials in the superpotential is simply the sum of the dimensions of each component. We are thus left unable to determine the quantum dimension of each term in (2.9) since this is a strongly coupled field theory. For the $\mathcal{N} = 8$ theory one can take k large and study perturbation theory however as mentioned already, the flows considered here are only supersymmetric for $k = 1, 2$.

On the other hand, if $m_7 = m_8$ one has $\mathcal{N} = 2$ supersymmetry and one can make a field-theory argument that the RG flow terminates at a CFT fixed point in the IR [19]. Indeed, when one integrates out the \mathcal{Z}^4 superfield one ends up with the superpotential [19]:

$$W_{\mathcal{N}=2} = \int d^3x d^2\theta (\epsilon_{abcd} \mathcal{Z}_1^a \mathcal{Z}_2^b \mathcal{Z}_3^c)^2. \quad (2.10)$$

At this point we have a $U(1)_R$ symmetry and thus we know that the dimension of all three complex fields \mathcal{Z}_A is given by:

$$\Delta_{\mathcal{Z}} = R_{\mathcal{Z}} = \frac{1}{3}. \quad (2.11)$$

As explained earlier, if one re-writes this $\mathcal{N} = 2$ theory and flow in terms of $\mathcal{N} = 1$ superfields then some terms in the $\mathcal{N} = 1$ superpotential come directly from the $\mathcal{N} = 2$ superpotential whilst others are related to the $\mathcal{N} = 2$ kinetic terms (D-terms). The former contain the relevant operators that drive the flow while the latter, being related to kinetic terms of fields that are frozen out, become irrelevant in the IR and are simply dropped.

One can use this perspective in thinking about flows with two non-zero and unequal masses, $m_7 \neq m_8$. There are various classes of monomials in the superpotential (2.7) before mass terms are added: terms can be independent of (Φ^7, Φ^8) , they can be linear or they can be quadratic in these fields:

$$\begin{aligned} \widetilde{W} = & g_{mn} g_{pq} \epsilon^{abcd} \widetilde{\mathcal{Z}}_a^m \widetilde{\mathcal{Z}}_b^n \widetilde{\mathcal{Z}}_c^p \widetilde{\mathcal{Z}}_d^q \\ & + \epsilon^{abcd} \left((\widetilde{\mathcal{Z}}_a^1 \widetilde{\mathcal{Z}}_b^2 \widetilde{\mathcal{Z}}_c^3 + \overline{\widetilde{\mathcal{Z}}}_a^1 \overline{\widetilde{\mathcal{Z}}}_b^2 \overline{\widetilde{\mathcal{Z}}}_c^3) \Phi_d^7 + i(\widetilde{\mathcal{Z}}_a^1 \widetilde{\mathcal{Z}}_b^2 \widetilde{\mathcal{Z}}_c^3 - \overline{\widetilde{\mathcal{Z}}}_a^1 \overline{\widetilde{\mathcal{Z}}}_b^2 \overline{\widetilde{\mathcal{Z}}}_c^3) \Phi_d^8 \right) \\ & + g_{mn} \epsilon^{abcd} \widetilde{\mathcal{Z}}_a^m \overline{\widetilde{\mathcal{Z}}}_b^n \Phi_c^7 \Phi_d^8. \end{aligned} \quad (2.12)$$

where $m, n, \dots = 1, 2, 3$, and g_{mn} is some Kähler metric. The quadratic terms, coming from the final line in (2.12), prevent one from integrating out both Φ^7 and Φ^8 analytically. However, it is precisely these quadratic terms in (Φ^7, Φ^8) that have the form $\widetilde{\mathcal{Z}}^4 \overline{\widetilde{\mathcal{Z}}}^4$ and that become irrelevant in the $\mathcal{N} = 2$ flow. It therefore seems reasonable to assume that

these may be dropped in the general family of flows. Indeed, if we ignore these terms and integrate out (Φ^7, Φ^8) in $\widetilde{W} + \Delta W_{BLG}$ we find:

$$\begin{aligned} \widehat{W} = & g_{mn}g_{pq}\epsilon^{abcd}\overline{\tilde{Z}}_a^m\tilde{Z}_b^n\overline{\tilde{Z}}_c^p\tilde{Z}_d^q \\ & + h_1(\epsilon^{abcd}(\tilde{Z}_a^1\tilde{Z}_b^2\tilde{Z}_c^3 + \overline{\tilde{Z}}_a^1\overline{\tilde{Z}}_b^2\overline{\tilde{Z}}_c^3))^2 + h_2(\epsilon^{abcd}(\tilde{Z}_a^1\tilde{Z}_b^2\tilde{Z}_c^3 - \overline{\tilde{Z}}_a^1\overline{\tilde{Z}}_b^2\overline{\tilde{Z}}_c^3))^2. \end{aligned} \quad (2.13)$$

This contains terms that are quartic as well as sextic and the two parameters, h_1 and h_2 , are the remnants of the mass parameters. We are unable to argue purely from the field theory that this should flow to a SCFT in the IR, however, the gravity dual suggests that it will flow to the $SU(3) \times U(1)$ symmetric $\mathcal{N}=2$ point. This implies that in the IR $h_1 = h_2$ and that the quartic terms become tied by $\mathcal{N}=2$ supersymmetry to the kinetic terms for the \tilde{Z}^m fields.

The main reason for not being able to provide an argument purely from the field theory for the phase structure of this family of flows is that in three dimensions, $\mathcal{N}=1$ supersymmetry has no \mathcal{R} -symmetry and thus no chiral ring. However the main feature of AdS/CFT is that the gravity dual can be used to study strongly coupled field theory, and for the class of field theories considered here we will see that the gravity dual provides much sharper information about the phase structure.

3 Mass perturbations in maximal supergravity

3.1 The scalars of gauged supergravity and their holographic duals

The $SU(3)$ -invariant sector of gauged supergravity was studied long ago in [2, 21, 23]. In terms of the complex 4-forms that parametrize the $E_{7(7)}/SU(8)$ of the maximal theory, this six-dimensional sector may be parametrized as follows. Following [2, 23], introduce complex coordinates, (z_1, z_2, z_3, z_4) on \mathbb{R}^8 and define the real forms:

$$\begin{aligned} J^\pm &\equiv \frac{i}{2} \left(\sum_{j=1}^3 dz_j \wedge d\bar{z}_j \right) \pm \frac{i}{2} dz_4 \wedge d\bar{z}_4, \\ F_1^+ &\equiv J^+ \wedge J^+, & F_1^- &\equiv J^- \wedge J^-, \\ F_2^+ + iF_3^+ &\equiv dz_1 \wedge dz_2 \wedge dz_3 \wedge dz_4, & F_2^- + iF_3^- &\equiv dz_1 \wedge dz_2 \wedge dz_3 \wedge d\bar{z}_4. \end{aligned} \quad (3.1)$$

The forms F_j^+ and F_j^- are, respectively, self-dual and anti-self dual. The $SO(8)$ of gauged supergravity acts on \mathbb{R}^8 as the vector representation and there is $SU(3)$ subgroup that leaves all these forms invariant. There are also two $U(1)$'s in $SO(8)$ that commute with this $SU(3)$ and rotate the $z_j \rightarrow e^{i\alpha} z_j$, $j = 1, 2, 3$ and $z_4 \rightarrow e^{i\beta} z_4$. These $U(1)$ actions can be used to set $F_3^\pm = 0$.

These six four-forms may be viewed as defining six scalar fields in $\mathcal{N}=8$ supergravity and, as a sub-manifold of $\frac{E_{7(7)}}{SU(8)}$, they live in the coset

$$\frac{SU(1,1)}{U(1)} \times \frac{SU(2,1)}{SU(2) \times U(1)}, \quad (3.2)$$

where F_1^\pm defines the tangents to the first manifold and F_2^\pm and F_3^\pm define the tangents on the second. We will parametrize the scalar manifolds using (complex) scalar fields by, $w_j, j = 1, 2, 3$, with the $E_{7(7)}$ components given by:

$$\Sigma = \sum_{j=1}^3 \left(\text{Re}(w_j) F_j^+ + i \text{Im}(w_j) F_j^- \right), \tag{3.3}$$

whose exponential form, in terms of the coset (3.2), reduces to:

$$\mathcal{M}_1 = \exp \begin{pmatrix} 0 & w_1 \\ \bar{w}_1 & 0 \end{pmatrix}, \quad \mathcal{M}_2 = \exp \begin{pmatrix} 0 & 0 & w_2 \\ 0 & 0 & w_3 \\ \bar{w}_2 & \bar{w}_3 & 0 \end{pmatrix}. \tag{3.4}$$

The gauged supergravity theory in four dimensions contains 70 scalar fields, and these are holographically dual to the (traceless) bilinears in the scalars and fermions:

$$\begin{aligned} \mathcal{O}^{IJ} &= \text{Tr} (X^I X^J) - \frac{1}{8} \delta^{IJ} \text{Tr} (X^K X^K), \quad I, J, \dots = 1, \dots, 8 \\ \mathcal{P}^{AB} &= \text{Tr} (\lambda^A \lambda^B) - \frac{1}{8} \delta^{AB} \text{Tr} (\lambda^C \lambda^C), \quad A, B, \dots = 1, \dots, 8, \end{aligned} \tag{3.5}$$

where \mathcal{O}^{IJ} transforms in the $\mathbf{35}_s$ of $\text{SO}(8)$, and \mathcal{P}^{AB} transforms in the $\mathbf{35}_c$. The real parts of w_j can be thought of as the duals of \mathcal{O}^{IJ} for $I, J = 7, 8$ and the imaginary parts of w_j can be thought of as the duals of \mathcal{P}^{IJ} for $I, J = 7, 8$. The real and imaginary parts of the scalar, w_1 , are separately invariant under distinct $\text{SU}(4) \times \text{U}(1)$ groups, which means that w_1 is dual to the following operator:

$$w_1 \leftrightarrow (\mathcal{O}^{77} + \mathcal{O}^{88}) + i(\mathcal{P}^{77} + \mathcal{P}^{88}). \tag{3.6}$$

Similarly, $F_1^+ + iF_2^+$ is dual to $\text{Tr}((X^7 + iX^8)^2)$ and $F_1^- + iF_2^-$ is dual to $\text{Tr}((\lambda^7 + i\lambda^8)^2)$. Thus

$$w_2 \leftrightarrow (\mathcal{O}^{77} - \mathcal{O}^{88}) + i(\mathcal{P}^{77} - \mathcal{P}^{88}), \quad w_3 \leftrightarrow \mathcal{O}^{78} + i\mathcal{P}^{78} \tag{3.7}$$

One can use the residual $\text{U}(1) \times \text{U}(1)$ invariance to diagonalize the fermion and boson mass matrices and take $w_3 = 0$. To get the G_2 invariant critical point and flows one takes $w_1 = \pm w_2, w_3 = 0$, while for the $\text{SU}(3)$ invariant critical point and flow one takes $\text{Im}(w_1) = \text{Re}(w_2) = w_3 = 0$ [2].

3.2 The scalar action of gauged supergravity

To exponentiate the scalar matrices, it is convenient to use a polar parametrization and take (for $w_3 = 0$):

$$w_1 = \lambda e^{-2i\phi}, \quad w_2 = \frac{1}{2} \chi e^{i\varphi}. \tag{3.8}$$

After exponentiating one can write the matrices (3.4) in terms of the scalar fields:

$$\zeta_1 = \tanh \lambda e^{-2i\phi}, \quad \zeta_2 = \tanh\left(\frac{1}{2}\chi\right) e^{i\varphi}, \tag{3.9}$$

for which the supergravity Lagrangian [24] gives the kinetic term:

$$\mathcal{L}_{kin.} = - \left[3 \frac{\nabla_\mu \zeta_1 \nabla^\mu \bar{\zeta}_1}{(1 - |\zeta_1|^2)^2} + 4 \sum_{j=2}^3 \frac{\nabla_\mu \zeta_j \nabla^\mu \bar{\zeta}_j}{(1 - (|\zeta_2|^2 + |\zeta_3|^3)^2)} \right], \quad (3.10)$$

where we have restored ζ_3 via symmetry. In terms of the polar representation one has:

$$\begin{aligned} \mathcal{L}_{kin.} &= -K_{ij} (\nabla_\mu \psi^i) (\nabla^\mu \psi^j) \\ &= -[(\partial_\mu \chi)^2 + \sinh^2 \chi (\partial_\mu \varphi)^2 + 3((\partial_\mu \lambda)^2 + \sinh^2 2\lambda (\partial_\mu \phi)^2)], \end{aligned} \quad (3.11)$$

where K_{ij} is the metric on the scalar space with $\psi^i = (\lambda, \chi, \phi, \varphi)$.

Following [6, 25–27],² a superpotential can be extracted from the eigenvalues of the A_1 -tensor that appears in the variation of the gravitino of the $\mathcal{N}=8$ theory [24]. In the $SU(3)$ invariant sector there are two candidate eigenvalues³ [3] that are related by $\zeta_2 \rightarrow -\bar{\zeta}_2$. Choosing one of these eigenvalues, we define the complex superpotential, \mathcal{W} , by

$$\mathcal{W} = (1 - |\zeta_1|^2)^{-\frac{3}{2}} (1 - |\zeta_2|^2)^{-2} \left[(1 + \zeta_1^3)(1 + \zeta_2^4) + 6 \zeta_1 \zeta_2^2 (1 + \zeta_1) \right]. \quad (3.12)$$

The supergravity potential on the $SU(3)$ invariant sector [2] is then given by [3]:

$$\begin{aligned} \mathcal{P} &= 2g^2 \left[\left| \frac{\partial \mathcal{W}}{\partial \chi} \right|^2 + \frac{4}{3} \left| \frac{\partial \mathcal{W}}{\partial \lambda} \right|^2 - 3 |\mathcal{W}|^2 \right] \\ &= 2g^2 \left[\frac{4}{3} (1 - |\zeta_1|^2)^2 \left| \frac{\partial \mathcal{W}}{\partial \zeta_1} \right|^2 + (1 - |\zeta_2|^2)^2 \left| \frac{\partial \mathcal{W}}{\partial \zeta_2} \right|^2 - 3 |\mathcal{W}|^2 \right]. \end{aligned} \quad (3.13)$$

The real superpotential is given by $|\mathcal{W}|$ and one also has:

$$\mathcal{P} = 2g^2 \left[\left(\frac{\partial |\mathcal{W}|}{\partial \chi} \right)^2 + \frac{1}{\sinh^2 \chi} \left(\frac{\partial |\mathcal{W}|}{\partial \varphi} \right)^2 + \frac{1}{3} \left(\frac{\partial |\mathcal{W}|}{\partial \lambda} \right)^2 + \frac{1}{3 \sinh^2 2\lambda} \left(\frac{\partial |\mathcal{W}|}{\partial \phi} \right)^2 - 3 |\mathcal{W}|^2 \right]. \quad (3.14)$$

This is a consequence of identities that come from the fact that \mathcal{W} is holomorphic up to an overall pre-factor:

$$\partial_\phi \log \mathcal{W} - i \sinh 2\lambda \partial_\lambda \log \left(\frac{\mathcal{W}}{|\mathcal{W}|} \right) = 0, \quad \partial_\varphi \log \mathcal{W} + i \sinh \chi \partial_\chi \log \left(\frac{\mathcal{W}}{|\mathcal{W}|} \right) = 0. \quad (3.15)$$

The superpotential has an $SO(8)$ -invariant critical point, with $\mathcal{N}=8$ supersymmetry, at $\zeta_1 = \zeta_2 = 0$ and with cosmological constant, $\Lambda = -6g^2$. The $SU(3) \times U(1)$ -invariant critical point, with $\mathcal{N}=2$ supersymmetry is given by:

$$\begin{aligned} \lambda &= \lambda_2 \equiv \frac{1}{4} \log(3), & \chi &= \pm \chi_2 \equiv \pm \log \left(\frac{\sqrt{3}-1}{\sqrt{2}} \right), \\ \phi &= \phi_2 \equiv 0, & \varphi &= \pm \varphi_2 \equiv \pm \frac{\pi}{2}; \\ \Lambda_{SU(3)} &= -\frac{9\sqrt{3}}{2} g^2 \approx -7.79423 g^2, \end{aligned} \quad (3.16)$$

²See also [28, 29] for relevant work on realizing holographic RG flows using gauged supergravity.

³The $SU(3) \times U(1)$ invariant sector is given by taking ζ_1 to be real and ζ_2 to be purely imaginary, and hence these two eigenvalues are equal.

with all possible choices of signs. In terms of the complex variables this corresponds to:

$$\zeta_1 = 2 - \sqrt{3}, \quad \zeta_2 = \pm i(\sqrt{3} - \sqrt{2}). \quad (3.17)$$

The G_2 -invariant critical point, with $\mathcal{N}=1$ supersymmetry is given by:

$$\begin{aligned} \lambda &= \pm \frac{1}{2}\chi = \frac{1}{2}\chi_1 \equiv \frac{1}{4} \log \left(\frac{1}{5} \left(1 + 4\sqrt{3} + 2\sqrt{2}\sqrt{3 + \sqrt{3}} \right) \right), \\ \phi &= -\frac{1}{2}\varphi = \phi_1 \equiv -\frac{1}{2}\varphi_1 \equiv \frac{1}{2} \arccos \left(\frac{\sqrt{3 - \sqrt{3}}}{2} \right); \\ \Lambda &= -\frac{216\sqrt{2}3^{\frac{1}{4}}}{25\sqrt{5}} g^2 \approx -7.19158 g^2. \end{aligned} \quad (3.18)$$

There is also a solution with $\chi \rightarrow \chi + \pi$. The G_2 critical points are given by $\zeta_2 = \pm \zeta_1^{\pm 1}$ for all choices of sign. The actual values of ζ_1 are a rather unedifying mess.

3.3 The supersymmetric flow equations

To set up a supersymmetric flow one takes the four-dimensional metric to have the form:

$$ds_{1,3}^2 = dr^2 + e^{2A(r)} (\eta_{\mu\nu} dx^\mu dx^\nu). \quad (3.19)$$

We take the Lagrangian of the scalars coupled to gravity to be:

$$\mathcal{L} = \frac{1}{2}R - \mathcal{P} + \mathcal{L}_{kin.} . \quad (3.20)$$

The supersymmetric flow equations are then obtained from the supersymmetry variations of the fermions and one finds [3]:

$$\begin{aligned} \frac{d\lambda}{dr} &= \pm \frac{\sqrt{2}g}{3} \partial_\lambda |\mathcal{W}|, & \frac{d\chi}{dr} &= \pm \sqrt{2}g \partial_\chi |\mathcal{W}|, \\ \frac{d\phi}{dr} &= \pm \frac{\sqrt{2}g}{3 \sinh^2 2\lambda} \partial_\phi |\mathcal{W}|, & \frac{d\varphi}{dr} &= \pm \frac{\sqrt{2}g}{\sinh^2 \chi} \partial_\varphi |\mathcal{W}|, & \frac{dA}{dr} &= \mp \sqrt{2}g |\mathcal{W}|. \end{aligned} \quad (3.21)$$

The equations for the flow of the scalars may be rewritten in terms of the scalar metric:

$$\frac{d\psi^i}{dr} = \pm \sqrt{2}g K^{ij} \frac{\partial |\mathcal{W}|}{\partial \psi^j}, \quad (3.22)$$

where K^{ij} is the inverse of the metric K_{ij} defined in (3.11). In terms of the complex coordinates (3.21) become

$$\frac{d\zeta_1}{dr} = \pm \frac{2\sqrt{2}g}{3} (1 - |\zeta_1|^2)^2 \frac{\partial \mathcal{W}}{\partial \bar{\zeta}_1}, \quad \frac{d\zeta_2}{dr} = \pm \frac{g}{\sqrt{2}} (1 - |\zeta_2|^2)^2 \frac{\partial \mathcal{W}}{\partial \bar{\zeta}_2}. \quad (3.23)$$

Given the cosmological constants of the three supersymmetric critical points, they suggest a possible flow, by steepest descent from the G_2 point to the $SU(3) \times U(1)$ point. One can see graphically that this is possible. The superpotential, $|\mathcal{W}|$, depends upon four variables and

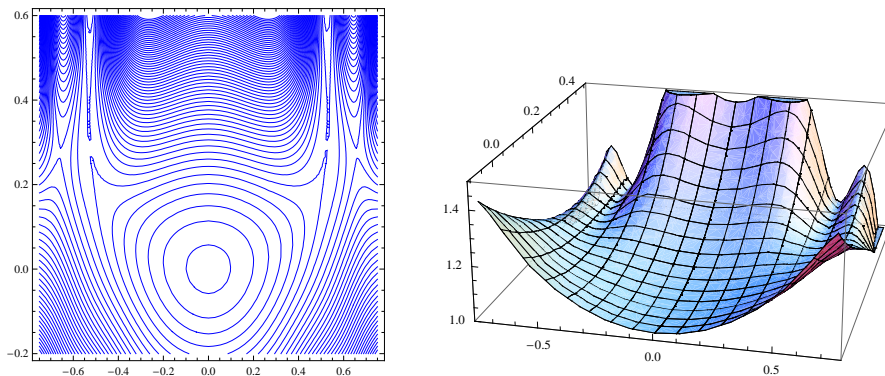


Figure 2. Plots of the function $|\widehat{\mathcal{W}}(\chi, \lambda)|$ obtained by making the substitutions (3.24) into the superpotential $|\mathcal{W}|$. The left-right axis is χ and the other axis is λ . There are five critical points visible and they are related by $\chi \rightarrow -\chi$. The SO(8) invariant critical point is the central minimum. Moving away from this, the first saddle points are the G_2 -invariant critical points and the second pair of highest saddles are the $SU(3) \times U(1)$ -invariant critical points.

the easiest way to see the critical points is to create a function of two variables, $|\widehat{\mathcal{W}}(\chi, \lambda)|$ by substituting the following into \mathcal{W} :

$$\phi = \phi_2 + (\phi_1 - \phi_2) \frac{(\chi^2 - \chi_2^2)}{(\chi_1^2 - \chi_2^2)}, \quad \varphi = \varphi_2 + (\varphi_1 - \varphi_2) \frac{(\chi^2 - \chi_2^2)}{(\chi_1^2 - \chi_2^2)}, \quad (3.24)$$

where the χ_j , ϕ_j and φ_j are defined in (3.18) and (3.16). This substitution ensures that the function $|\widehat{\mathcal{W}}|$ slices through the critical points of $|\mathcal{W}|$. The result is depicted in figure 2.

There is a unique steepest descent on the superpotential $|\mathcal{W}|$ that goes from the $SU(3) \times U(1)$ -invariant critical point to the G_2 invariant critical point. One can also find a family of steepest descent flows on $|\mathcal{W}|$ starting from the $SU(3) \times U(1)$ -invariant critical point and descending ultimately to the SO(8)-invariant critical point. There is the direct descent, which preserves $SU(3) \times U(1)$, and there are descents that approach the G_2 invariant fixed point first before turning down to the SO(8)-invariant critical point. Indeed one may approach G_2 invariant fixed point arbitrarily closely. These are depicted in figure 3.

The field theory flows are, of course, steepest descents on $-|\mathcal{W}|$ and therefore flow in the opposite direction to the foregoing discussion. The G_2 flow corresponds to tuning $m_1 \neq 0, m_2 = 0$, while the $SU(3) \times U(1)$ invariant flow corresponds to $m_1 = m_2$. From the supergravity it is evident that if one has the G_2 flow with $m_1 \neq 0$ and if one turns on a small value for m_2 , then the flow is deflected to the $SU(3) \times U(1)$ invariant fixed point and so m_2 grows until $m_2 = m_1$. Generic flows out of all of the fixed points typically run off to infinity, or “Hades,” and this simply means that the Coulomb branch is dominating the infra-red end of the flow [25, 30]. The interesting new feature is that there is a “cone,” or family, of flows bounded by the $SU(3) \times U(1)$ and G_2 invariant flows and whose infra-red limit is the $SU(3) \times U(1)$ invariant fixed point.

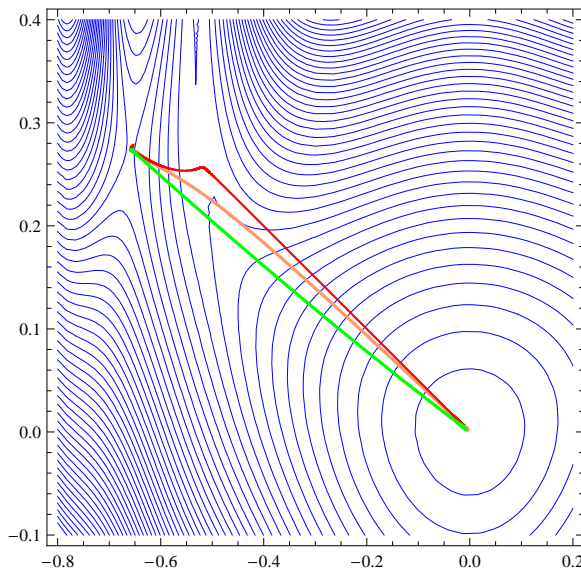


Figure 3. This shows details of the contour plot of $|\widehat{\mathcal{W}}|$ in figure 2. Three steepest descent paths shown: One going directly from the $SU(3) \times U(1)$ -invariant critical point to the $SO(8)$ -invariant critical point. Another goes from the $SU(3) \times U(1)$ -invariant critical point and passes extremely close to the G_2 -invariant critical point before descending to the $SO(8)$ -invariant critical point. The third is a generic intermediate path between these extremes. The physical holographic RG flows follow these trajectories in reverse. Note also that there is some relative distortion of the paths and the contours because the paths represent numerical solutions on the complete superpotential, $|\mathcal{W}|$, while the contours are those of $|\widehat{\mathcal{W}}|$.

3.4 Flows near the critical points

To understand the pattern of the flows around the three fixed points, it is instructive to compute the scaling dimensions of the operators in the $SU(3)$ -invariant sectors and see how they govern the flows. This requires the linearization of the flow equations in the neighborhood of the fixed points.

For the $SO(8)$ -invariant fixed point, the polar coordinate system is singular and it is more convenient to linearize (3.23) which, around $\zeta_j = 0$, give:

$$\frac{d\zeta_1}{dr} \approx \pm \sqrt{2} g \zeta_1 + \dots, \quad \frac{d\zeta_2}{dr} \approx \pm \sqrt{2} g \zeta_2 + \dots, \quad \frac{dA}{dr} \approx \mp \sqrt{2} g + \dots \quad (3.25)$$

The canonical form of the AdS metric of radius L is to take:

$$A(r) = e^{r/L}, \quad (3.26)$$

which means that modes are non-normalizable if they behave as:

$$e^{-\Delta r/L}, \quad \Delta \leq \frac{3}{2}. \quad (3.27)$$

For the flows (3.25) one has:

$$g = \mp \frac{1}{\sqrt{2} L}, \quad \zeta_1 = a_1 e^{-r/L}, \quad \zeta_2 = a_2 e^{-r/L}. \quad (3.28)$$

for some constants a_j , and so these modes are all non-normalizable. Thus they represent mass insertions into the Lagrangian and not vevs of background fields. There is an ambiguity in the holographic dictionary if a field that has dimension Δ has a supergravity mode that scales as:

$$e^{-\Delta r/L} \quad \text{or} \quad e^{-(3-\Delta)r/L}. \quad (3.29)$$

For the fermionic and bosonic mass terms one has $\Delta = 2$ and $\Delta = 1$ and these correspond to (3.28) provided that the fermions and bosons correspond to different choices in (3.29).

One expects terms in the Lagrangian that are related by $\mathcal{N}=1$ supersymmetry to have scaling dimensions that differ by 1. One should note that, when this is translated through the foregoing holographic dictionary, the dual supergravity scalars in a supermultiplet are expected to have exponents that either differ by 1 or that sum to 2. The exponents in (3.28) have the latter behavior.

In the neighborhood of the other two fixed points it is convenient to use the polar form of the Lagrangian and linearize (3.22). Indeed, one obtains the canonical AdS metric, (3.26) if one now takes

$$g = \mp \frac{1}{\sqrt{2} L_* |\mathcal{W}|_*}, \quad (3.30)$$

where $|\mathcal{W}|_*$ is the value of the superpotential at the critical point and L_* is the AdS radius corresponding to the fixed point. The linearization of (3.22) is then

$$\frac{d\psi^i}{dr} = -\frac{1}{L_*} \mathcal{M}^i_k (\psi^k - \psi_0^k), \quad \mathcal{M}^i_k \equiv \left(\frac{K^{ij}}{|\mathcal{W}|} \frac{\partial^2 |\mathcal{W}|}{\partial \psi^j \partial \psi^k} \right)_*, \quad (3.31)$$

where $*$ denotes the value at the critical point. Therefore, we need the eigenvalues of the matrix \mathcal{M}^i_k .

At the $SU(3) \times U(1)$ invariant fixed point the eigenvalues of \mathcal{M}^i_k are:

$$\left(\frac{1}{2}(1 - \sqrt{17}), \frac{1}{2}(3 - \sqrt{17}), \frac{1}{2}(\sqrt{17} + 1), \frac{1}{2}(\sqrt{17} + 3) \right) \\ \approx (-1.56155, -0.561553, 2.56155, 3.56155), \quad (3.32)$$

and at the G_2 invariant fixed point the eigenvalues of \mathcal{M}^i_k are:

$$\left(1 - \sqrt{6}, 1 - \frac{1}{\sqrt{6}}, \left(\frac{1}{\sqrt{6}} + 1 \right), (\sqrt{6} + 1) \right) \\ \approx (-1.44949, 0.591752, 1.40825, 3.44949). \quad (3.33)$$

Note that at each point the eigenvalues come in pairs that add to 2, consistent with $\mathcal{N}=1$ supersymmetry. Negative eigenvalues correspond to irrelevant operators that flow into the fixed point in the infra-red. There is one such operator for the G_2 -invariant point, corresponding to the flow from the $SO(8)$ invariant fixed point. There are two such operators for the $SU(3) \times U(1)$ point and these correspond to the family, or cone, of flows that arrive at the $SU(3) \times U(1)$ point from the $SO(8)$ point. The positive eigenvalues correspond to relevant operators or to vevs that drive the flow away from the fixed point in the infra-red. The two positive eigenvalues at the $SU(3) \times U(1)$ point are greater than

$\frac{3}{2}$, which means the modes are normalizable and therefore correspond to perturbations of the state of the system. Based upon the experience of [25, 27, 30], it seems reasonable to expect that they correspond to some form of Coulomb branch flow.

At the G_2 point there are three positive eigenvalues. One of them is normalizable and presumably corresponds to a Coulomb branch flow but, in contrast to the analogous situation in four-dimensional Yang-Mills theory, the scaling dimension is greater than 3 and so this “Coulomb flow” is being driven by the vev of an irrelevant operator. More interesting are the two other eigenvalues, $1 \pm \frac{1}{\sqrt{6}}$, which correspond to non-normalizable modes, and hence must represent perturbations of the Lagrangian. Note that these eigenvalues sum to 2 and thus may be interpreted as supersymmetric counterparts of one another. Indeed, they must represent the fermionic and bosonic mass terms that generate the $\mathcal{N} = 1$ supersymmetric flow from the G_2 point to the $SU(3) \times U(1)$ point. Supergravity therefore predicts the dimensions of the corresponding operator to be $1 \pm \frac{1}{\sqrt{6}}$ and $2 \pm \frac{1}{\sqrt{6}}$, and hence there is an anomalous dimension of $\pm \frac{1}{\sqrt{6}}$. It is interesting that the dimensions are not rational, but this is entirely possible since there is no continuous \mathcal{R} -symmetry to protect operator dimensions. It would be most interesting to see if there is a way to compute these relevant operator dimensions directly within the field theory.

4 Final comments

We have studied the field theory on a stack of membranes by deforming the theory with mass terms. The specific mass terms we considered trigger flows that terminate at superconformal Chern-Simons matter theories in the IR. The phase structure of the general flow in this class is hard to study directly in the field theory since only $\mathcal{N} = 1$ supersymmetry is preserved. Nevertheless our study of the gravity dual provides a compelling description of these flows.

The remaining challenges directly related to this family of flows lie in the field theory. For instance, simply calculating the dimension of operators at the G_2 symmetric point and comparing them to the supergravity spectrum would be an important achievement since there is no holomorphy in the field theory and it is strongly coupled.

There are many other $\mathcal{N} = 1$ supersymmetric mass terms that can be considered and it would be interesting to study these using holography. One particular class of these flows involves an equal mass term for all four complex scalars and was considered from the gravity point of view [31–33] and from the field theory point of view [34, 35]. A related, non-holomorphic mass deformation was studied in [36]. This flow preserves sixteen supercharges and has a number of isolated vacua; it remains unsolved how to count these vacua correctly from the field theory. The difficulty in studying these mass deformations in the ABJM model is the same difficulty we have encountered in the current work, namely that the mass terms preserve the supersymmetry which is not manifest in the ABJM model.

There are also flows with equal mass terms for two complex scalars and preserving eight supersymmetries. These have been studied in [37, 38] and can be considered as the analogue of the $\mathcal{N} = 2^*$ mass deformation of $\mathcal{N} = 4$ SYM in four dimensions [39], which flows to large- N Donagi-Witten theory [40]. Another family of flows that has not been examined closely

in supergravity is the deformation with equal masses for three complex scalars. This should also preserve $SU(3) \times U(1)$ symmetry but should not terminate at an SCFT. This high level of symmetry should also be sufficient to make it amenable to study from the supergravity perspective, perhaps even calculating the full eleven-dimensional solution. Moreover, the corresponding field theory should be related to the compactification to three dimensions of $\mathcal{N} = 1^*$ Yang-Mills theory, obtained by giving masses to the three chiral multiplets in $\mathcal{N} = 4$ Yang-Mills theory. The corresponding field theory in $(2 + 1)$ dimensions has been extensively studied and used to compute exact elliptic superpotentials [41, 42]. Given the new developments in the field theory on the M2 branes it would be very interesting to revisit these earlier results and see how they are related via massive flows.

It would also be very interesting to uplift the RG flow solutions that we found in four-dimensional gauged supergravity to eleven dimensions. This has been already done for the $\mathcal{N} = 2$ flow which corresponds to $m_1 = m_2$ in [8]. It is well known how to uplift the metric of solutions to four-dimensional $\mathcal{N} = 8$ gauged supergravity to eleven dimensions [43], however the techniques for finding the internal fluxes are rather cumbersome [44]. One of the non-trivial features of the solution in [8] is the presence of internal four-form flux and one can expect that such flux will be present for the whole $SU(3)$ invariant family of flow solutions discussed here. The solutions with $m_1 \neq m_2$ will have also smaller internal symmetry group and less supersymmetry which makes the eleven-dimensional uplift a non-trivial task.

In terms of string compactifications, AdS_4 vacua are phenomenologically interesting for many reasons. It would be interesting to develop a better understanding of such backgrounds which preserve only two supercharges and the dual three-dimensional field theory is presumably a useful place to perform such studies. As such, $\mathcal{N} = 1$ CS-matter theories, like the ones studied in this paper, may be an appropriate place to start.

Acknowledgments

NH would like to thank Michela Petrini for discussions. The work of NB, KP and NPW is supported in part by the DOE grant DE-FG03-84ER-40168. The work of NH is supported by DSM CEA-Saclay and the grant Marie Curie IRG 046430. NB is supported also by a Graduate Fellowship from KITP and in part by the National Science Foundation under Grant No. PHY05-51164.

A Projecting $\mathcal{N} = 2$ to $\mathcal{N} = 1$ superspace in three dimensions

Here we summarize some aspects of how to break up the three-dimensional $\mathcal{N} = 2$ superfields into $\mathcal{N} = 1$ superfields, a complete description is given in [45, 46]. This is useful as it allows one to consider an $\mathcal{N} = 2$ action and add $\mathcal{N} = 1$ preserving operators to it.

The complex spinor of $\mathcal{N} = 2$ superspace is decomposed as⁴ $\theta = \theta_1 + i\theta_2$ and to reduce the action to $\mathcal{N} = 1$ supersymmetry we integrate out the θ_2 dependence. First we

⁴The irreducible spinor θ^α in three dimensions has two real components but often its complex counterpart with four real components is also denoted θ^α . We hope this will not cause too much confusion.

decompose the $N = 2$ differentials in terms of $\mathcal{N} = 1$ differentials

$$D_\alpha = \frac{1}{2}(D_{1\alpha} + iD_{2\alpha}), \quad \overline{D}^\alpha = \frac{1}{2}(D_1^\alpha - iD_2^\alpha). \quad (\text{A.1})$$

This allows us to write the superspace $\mathcal{N} = 2$ measures in a way which then facilitates the reduction of the action to $\mathcal{N} = 1$ superspace

$$\int d^3x d^4\theta = - \int d^3x D_1^2 D_2^2, \quad \int d^3x d^2\theta = \int d^3x D_1^2. \quad (\text{A.2})$$

Then the $\mathcal{N} = 2$ fields reduce to $\mathcal{N} = 1$ fields as

$$\mathcal{Z}|_{\theta_2=0} = \tilde{\mathcal{Z}}, \quad \overline{\mathcal{Z}}|_{\theta_2=0} = \overline{\tilde{\mathcal{Z}}}, \quad \mathcal{V}|_{\theta_2=0} = 0, \quad D_{2\alpha}\mathcal{V}|_{\theta_2=0} = \Gamma_\alpha, \quad D_2^2\mathcal{V}|_{\theta_2=0} = R \quad (\text{A.3})$$

where $\tilde{\mathcal{Z}}$ is a *complex* $\mathcal{N} = 1$ scalar superfield and R is *real* $\mathcal{N} = 1$ scalar superfield. Since $\mathcal{N} = 1$ superspace is real, we can break a complex scalar superfield into real and imaginary parts and in section 2 we used the complex structure

$$\tilde{\mathcal{Z}}^1 = \Phi^1 + i\Phi^2, \quad \tilde{\mathcal{Z}}^2 = \Phi^3 + i\Phi^4, \quad \tilde{\mathcal{Z}}^3 = \Phi^5 + i\Phi^6, \quad \tilde{\mathcal{Z}}^4 = \Phi^7 + i\Phi^8 \quad (\text{A.4})$$

where Φ^i are real $\mathcal{N} = 1$ superfields.

In three dimensions, the $\mathcal{N} = 2$ gauge superfield \mathcal{V} is a bosonic superfield while the $\mathcal{N} = 1$ gauge superfield Γ^α is a fermi superfield. In the CS matter theories studied in this paper, integrating out the auxiliary superfield R will result in additional $\mathcal{N} = 1$ superpotential terms.

References

- [1] N. Seiberg, *Notes on theories with 16 supercharges*, *Nucl. Phys. Proc. Suppl.* **67** (1998) 158 [[hep-th/9705117](#)] [[SPIRES](#)].
- [2] N.P. Warner, *Some new extrema of the scalar potential of gauged $N = 8$ supergravity*, *Phys. Lett.* **B 128** (1983) 169 [[SPIRES](#)].
- [3] C.-h. Ahn and K. Woo, *Supersymmetric domain wall and RG flow from 4-dimensional gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 599** (2001) 83 [[hep-th/0011121](#)] [[SPIRES](#)].
- [4] C.-H. Ahn and K.-S. Woo, *Domain wall and membrane flow from other gauged $D = 4$, $N = 8$ supergravity. I*, *Nucl. Phys.* **B 634** (2002) 141 [[hep-th/0109010](#)] [[SPIRES](#)].
- [5] C.-H. Ahn and K.-S. Woo, *Domain wall from gauged $D = 4$, $N = 8$ supergravity. II*, *JHEP* **11** (2003) 014 [[hep-th/0209128](#)] [[SPIRES](#)].
- [6] C.-h. Ahn and J. Paeng, *Three-dimensional SCFTs, supersymmetric domain wall and renormalization group flow*, *Nucl. Phys.* **B 595** (2001) 119 [[hep-th/0008065](#)] [[SPIRES](#)].
- [7] C.-h. Ahn and T. Itoh, *An $N = 1$ supersymmetric G_2 -invariant flow in M-theory*, *Nucl. Phys.* **B 627** (2002) 45 [[hep-th/0112010](#)] [[SPIRES](#)].
- [8] R. Corrado, K. Pilch and N.P. Warner, *An $N = 2$ supersymmetric membrane flow*, *Nucl. Phys.* **B 629** (2002) 74 [[hep-th/0107220](#)] [[SPIRES](#)].
- [9] C. Ahn, *Holographic supergravity dual to three dimensional $N = 2$ gauge theory*, *JHEP* **08** (2008) 083 [[arXiv:0806.1420](#)] [[SPIRES](#)].

- [10] C. Ahn, *Towards holographic gravity dual of $N = 1$ superconformal Chern-Simons gauge theory*, *JHEP* **07** (2008) 101 [[arXiv:0806.4807](#)] [[SPIRES](#)].
- [11] C. Ahn, *Comments on holographic gravity dual of $N = 6$ superconformal Chern-Simons gauge theory*, *JHEP* **03** (2009) 107 [[arXiv:0812.4363](#)] [[SPIRES](#)].
- [12] J.H. Schwarz, *Superconformal Chern-Simons theories*, *JHEP* **11** (2004) 078 [[hep-th/0411077](#)] [[SPIRES](#)].
- [13] J. Bagger and N. Lambert, *Modeling multiple $M2$'s*, *Phys. Rev. D* **75** (2007) 045020 [[hep-th/0611108](#)] [[SPIRES](#)].
- [14] A. Gustavsson, *Algebraic structures on parallel $M2$ -branes*, *Nucl. Phys. B* **811** (2009) 66 [[arXiv:0709.1260](#)] [[SPIRES](#)].
- [15] J. Bagger and N. Lambert, *Gauge Symmetry and Supersymmetry of Multiple $M2$ -Branes*, *Phys. Rev. D* **77** (2008) 065008 [[arXiv:0711.0955](#)] [[SPIRES](#)].
- [16] J. Bagger and N. Lambert, *Comments on multiple $M2$ -branes*, *JHEP* **02** (2008) 105 [[arXiv:0712.3738](#)] [[SPIRES](#)].
- [17] M. Van Raamsdonk, *Comments on the Bagger-Lambert theory and multiple $M2$ -branes*, *JHEP* **05** (2008) 105 [[arXiv:0803.3803](#)] [[SPIRES](#)].
- [18] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, $M2$ -branes and their gravity duals*, *JHEP* **10** (2008) 091 [[arXiv:0806.1218](#)] [[SPIRES](#)].
- [19] M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence*, *JHEP* **09** (2008) 072 [[arXiv:0806.1519](#)] [[SPIRES](#)].
- [20] I. Klebanov, T. Klose and A. Murugan, *AdS_4/CFT_3 — Squashed, stretched and warped*, *JHEP* **03** (2009) 140 [[arXiv:0809.3773](#)] [[SPIRES](#)].
- [21] H. Nicolai and N.P. Warner, *The $SU(3) \times U(1)$ invariant breaking of gauged $N = 8$ supergravity*, *Nucl. Phys. B* **259** (1985) 412 [[SPIRES](#)].
- [22] A. Mauri and A.C. Petkou, *An $N = 1$ superfield action for $M2$ branes*, *Phys. Lett. B* **666** (2008) 527 [[arXiv:0806.2270](#)] [[SPIRES](#)].
- [23] N.P. Warner, *Some properties of the scalar potential in gauged supergravity theories*, *Nucl. Phys. B* **231** (1984) 250 [[SPIRES](#)].
- [24] B. de Wit and H. Nicolai, *$N = 8$ supergravity*, *Nucl. Phys. B* **208** (1982) 323 [[SPIRES](#)].
- [25] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c -theorem*, *Adv. Theor. Math. Phys.* **3** (1999) 363 [[hep-th/9904017](#)] [[SPIRES](#)].
- [26] N.P. Warner, *Renormalization group flows from five-dimensional supergravity*, *Class. Quant. Grav.* **17** (2000) 1287 [[hep-th/9911240](#)] [[SPIRES](#)].
- [27] A. Khavaev and N.P. Warner, *A class of $N = 1$ supersymmetric RG flows from five-dimensional $N = 8$ supergravity*, *Phys. Lett. B* **495** (2000) 215 [[hep-th/0009159](#)] [[SPIRES](#)].
- [28] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics*, *JHEP* **12** (1998) 022 [[hep-th/9810126](#)] [[SPIRES](#)].

- [29] J. Distler and F. Zamora, *Non-supersymmetric conformal field theories from stable Anti-de Sitter spaces*, *Adv. Theor. Math. Phys.* **2** (1999) 1405 [[hep-th/9810206](#)] [[SPIRES](#)].
- [30] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, *JHEP* **07** (2000) 038 [[hep-th/9906194](#)] [[SPIRES](#)].
- [31] I. Bena, *The M-theory dual of a 3 dimensional theory with reduced supersymmetry*, *Phys. Rev. D* **62** (2000) 126006 [[hep-th/0004142](#)] [[SPIRES](#)].
- [32] C.N. Pope and N.P. Warner, *A dielectric flow solution with maximal supersymmetry*, *JHEP* **04** (2004) 011 [[hep-th/0304132](#)] [[SPIRES](#)].
- [33] I. Bena and N.P. Warner, *A harmonic family of dielectric flow solutions with maximal supersymmetry*, *JHEP* **12** (2004) 021 [[hep-th/0406145](#)] [[SPIRES](#)].
- [34] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, *JHEP* **10** (2004) 025 [[hep-th/0409174](#)] [[SPIRES](#)].
- [35] J. Gomis, A.J. Salim and F. Passerini, *Matrix theory of type IIB plane wave from membranes*, *JHEP* **08** (2008) 002 [[arXiv:0804.2186](#)] [[SPIRES](#)].
- [36] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, *A massive study of M2-brane proposals*, *JHEP* **09** (2008) 113 [[arXiv:0807.1074](#)] [[SPIRES](#)].
- [37] C.N. Gowdigere and N.P. Warner, *Flowing with eight supersymmetries in M-theory and F-theory*, *JHEP* **12** (2003) 048 [[hep-th/0212190](#)] [[SPIRES](#)].
- [38] C.N. Gowdigere, D. Nemeschansky and N.P. Warner, *Supersymmetric solutions with fluxes from algebraic Killing spinors*, *Adv. Theor. Math. Phys.* **7** (2004) 787 [[hep-th/0306097](#)] [[SPIRES](#)].
- [39] K. Pilch and N.P. Warner, *$N = 2$ supersymmetric RG flows and the IIB dilaton*, *Nucl. Phys. B* **594** (2001) 209 [[hep-th/0004063](#)] [[SPIRES](#)].
- [40] R. Donagi and E. Witten, *Supersymmetric Yang-Mills theory and integrable systems*, *Nucl. Phys. B* **460** (1996) 299 [[hep-th/9510101](#)] [[SPIRES](#)].
- [41] N. Dorey, *An elliptic superpotential for softly broken $N = 4$ supersymmetric Yang-Mills theory*, *JHEP* **07** (1999) 021 [[hep-th/9906011](#)] [[SPIRES](#)].
- [42] N. Dorey and S.P. Kumar, *Softly-broken $N = 4$ supersymmetry in the large- N limit*, *JHEP* **02** (2000) 006 [[hep-th/0001103](#)] [[SPIRES](#)].
- [43] B. de Wit, H. Nicolai and N.P. Warner, *The embedding of gauged $N = 8$ supergravity into $D = 11$ supergravity*, *Nucl. Phys. B* **255** (1985) 29 [[SPIRES](#)].
- [44] B. de Wit and H. Nicolai, *The consistency of the S^7 truncation in $D = 11$ supergravity*, *Nucl. Phys. B* **281** (1987) 211 [[SPIRES](#)].
- [45] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, *Renormalizations in supersymmetric and non-supersymmetric nonAbelian Chern-Simons field theories with matter*, *Nucl. Phys. B* **391** (1993) 333 [[SPIRES](#)].
- [46] L.V. Avdeev, G.V. Grigorev and D.I. Kazakov, *Renormalizations in Abelian Chern-Simons field theories with matter*, *Nucl. Phys. B* **382** (1992) 561 [[SPIRES](#)].